International Space Sciences School
Heliospheric physical processes for understanding Solar Terrestrial Relations
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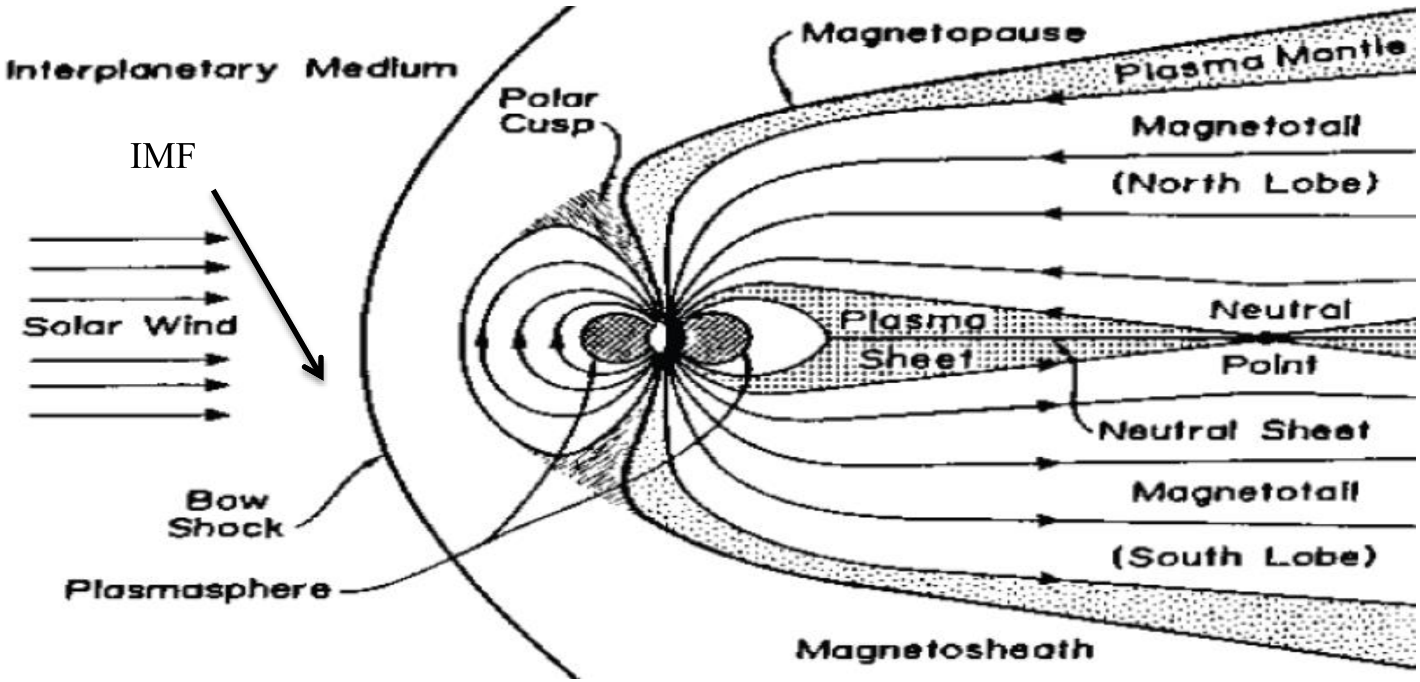
George K. Parks,
Space Sciences Laboratory, UC Berkeley, Berkeley, CA

Lecture 1: Introduction to Space Plasma Physics

A schematic diagram of the solar terrestrial plasma environment.



Understanding this diagram requires knowledge about

- transport of solar magnetic field
- how SW interacts with planetary magnetic fields
- collisionless shocks form
- the importance of neutral points
- current sheets
- particles acceleration mechanisms in the Magnetosphere.

Goal of the Lectures:

- To help understand space plasma behavior*, we will provide useful material* not normally found in space plasms textbooks (Russell and Kivelson, 1996; Parks, 1996; 2004) .

- Space plasma features are complex and often can have more than one interpretation.

- *Identify Issues with some models* and suggest different ways to interpret the data or how to resolve the issues.

- You *may not agree* with the ideas and concepts given in these lectures. *Criticisms, comments and questions* are welcome!

Point of View:

- Information about space plasmas comes from measurements made by in-situ experiments. If there are disagreement about interpretation, we go back to data.

- This first lecture will briefly review

(1) how detectors work and what they measure,

(2) what assumptions are made in the measurements, which affect interpretation of data,

(3) basic plasma theories and concepts needed to interpret the data.

Plasma Instrument and Measurements

- Focus on space plasmas with energies a few eV to ~40 keV/charge which includes most of solar wind and magnetospheric plasmas.

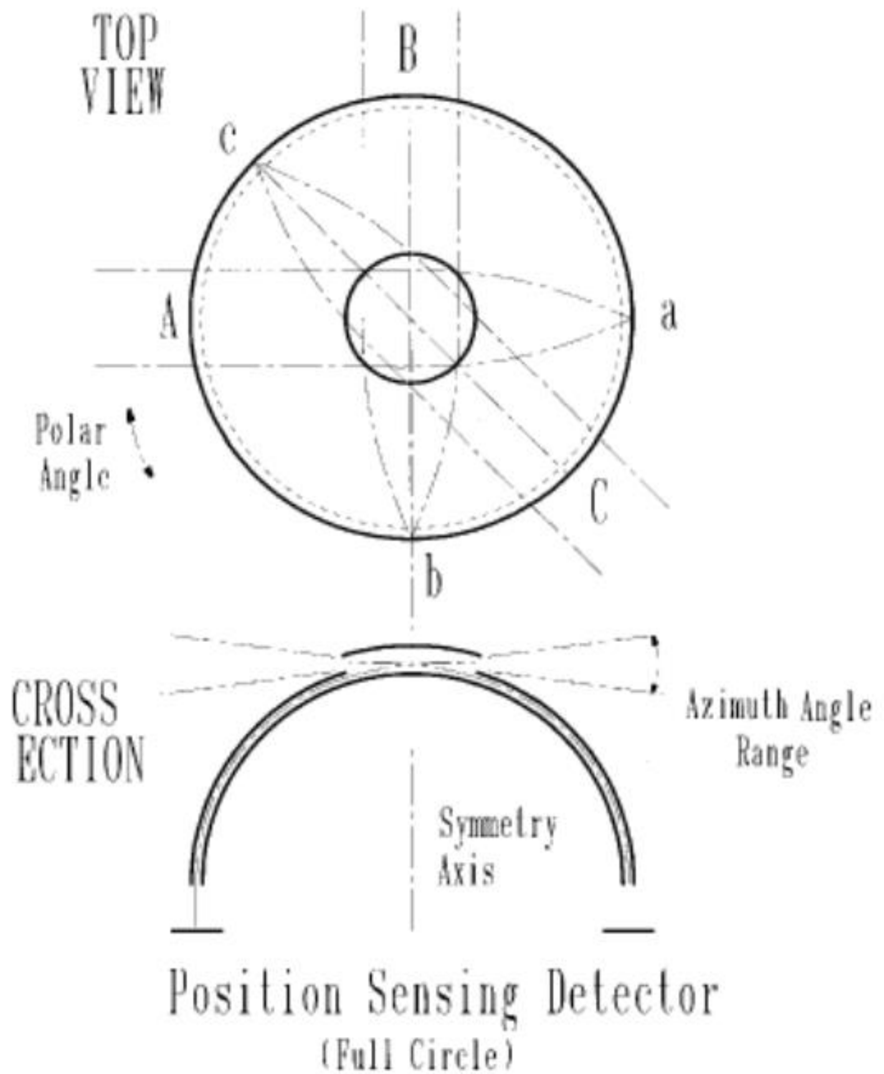
- Instrument most commonly used are ESAs, Faraday cups and SSDs.

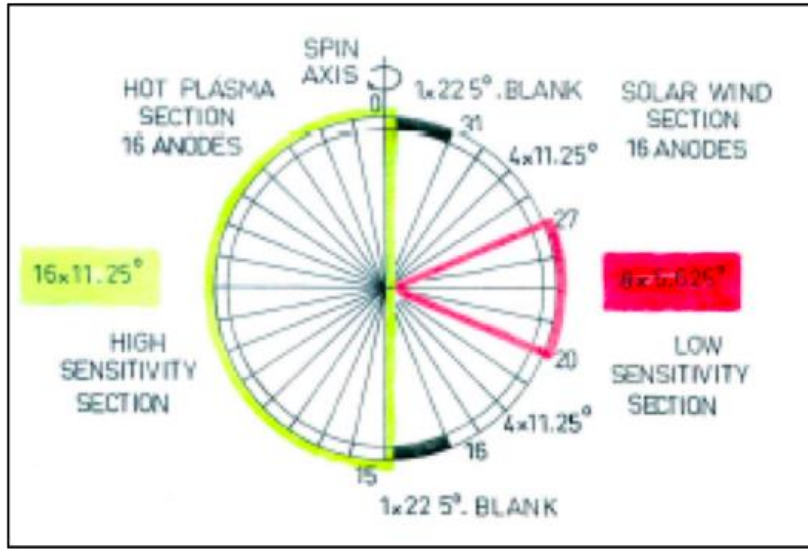
- ESAs and FCs are *energy/charge detectors*.

- Solid state detectors are total energy detectors, mainly used for detecting higher energy particles. *New* SSDs can measure particles from *a few keV* to several MeV and higher.

- We limit discussion of how ESAs work (See Wüest et al., 2007 for other types of detectors)

Cluster and Wind Ion instruments





- A schematic diagram of a symmetric spherical “top hat" ESA (Carlson et al., 1987).
- 3D information obtained in one spin of the spacecraft.
- Cluster and Wind instrument (Lin et al., 1995; Réme et al., 1997).

• Concentric spheres have a mean radius R. electric field E applied between the plates.

• Particles travel in circular path will pass through the plates only if the *electric force just balances the centripetal force*,

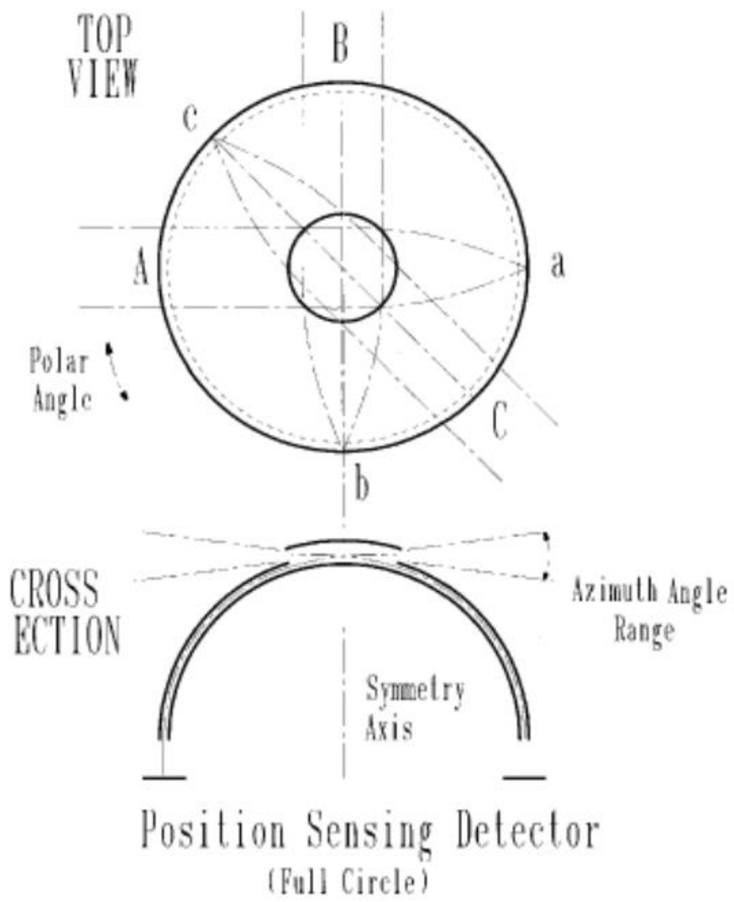
$$mv^2/R=qE$$

• Rewrite this equation,

$$mv^2/2q = ER/2$$

where *energy/charge* (left side) is related to instrument quantities (voltage & radius) on the right.

• ESAs measure *energy/charge* of the particle, regardless of the mass, charge or velocity.



Important FACT:

• Immerse ESA in plasma of average density n where the particles move with a mean velocity $\langle \mathbf{v} \rangle$. Total particle flux entering the aperture of a detector is $n\langle \mathbf{v} \rangle$, where

$$n(\mathbf{r}) = \int f(\mathbf{r}, \mathbf{v})\, d^3v$$

$$\langle \mathbf{v} \rangle = \int \mathbf{v}\, f(\mathbf{r}, \mathbf{v})\, d^3v$$

Here $f(\mathbf{r}, \mathbf{v})$ is the distribution function of the particles.

• A detector measures the product $n \langle \mathbf{v} \rangle$, ***not*** n or $\langle \mathbf{v} \rangle$.

- *A detector* counts particles. The total count is

$$C = n\langle \boldsymbol{v} \rangle A\, \Delta t,$$

where A is the effective area of the entrance aperture and Δt is the accumulation time.

- Energy/charge *(E/q) spectrum* is obtained by measuring the particles over a small energy range ΔE (Note E used for both electric field and energy).

- Define a *differential number of counts:*

$$C_i = n_i \langle \boldsymbol{v} \rangle_i\, A\, \Delta E_i\, \Delta t$$

where C_i represents counts in the HV step i covering the narrow energy range ΔE_i.

- Total *E/q spectrum* over the entire energy range obtained by varying the high voltage (HV) applied between the plates. The number of energy steps for a typical ESA is 16, 32 or 64.

- *Differential number flux*

$$F_N = C/g_v\, E\, \Delta t = C/g_E\, E\, \Delta t$$

Units: (cm^{-2}-s^{-1}-sr^{-1}-eV^{-1}), g_E (cm^2-sr), E in eV or keV.

- *Energy flux*

$$F_E = C/g_E \Delta t \qquad \textit{Units: } ergs/cm^2\text{-}s$$

- *Distribution function*

$$f(v) = C/\Delta t\, g_E\, v^4 \qquad \text{Units: } s^3\text{-}cm^{-6}.$$

F_N, F_E, *and* $f(\mathbf{r},\mathbf{v})$ are *primary quantities* measured by instruments.

Macroscopic quantities are *computed* from measured quantities:

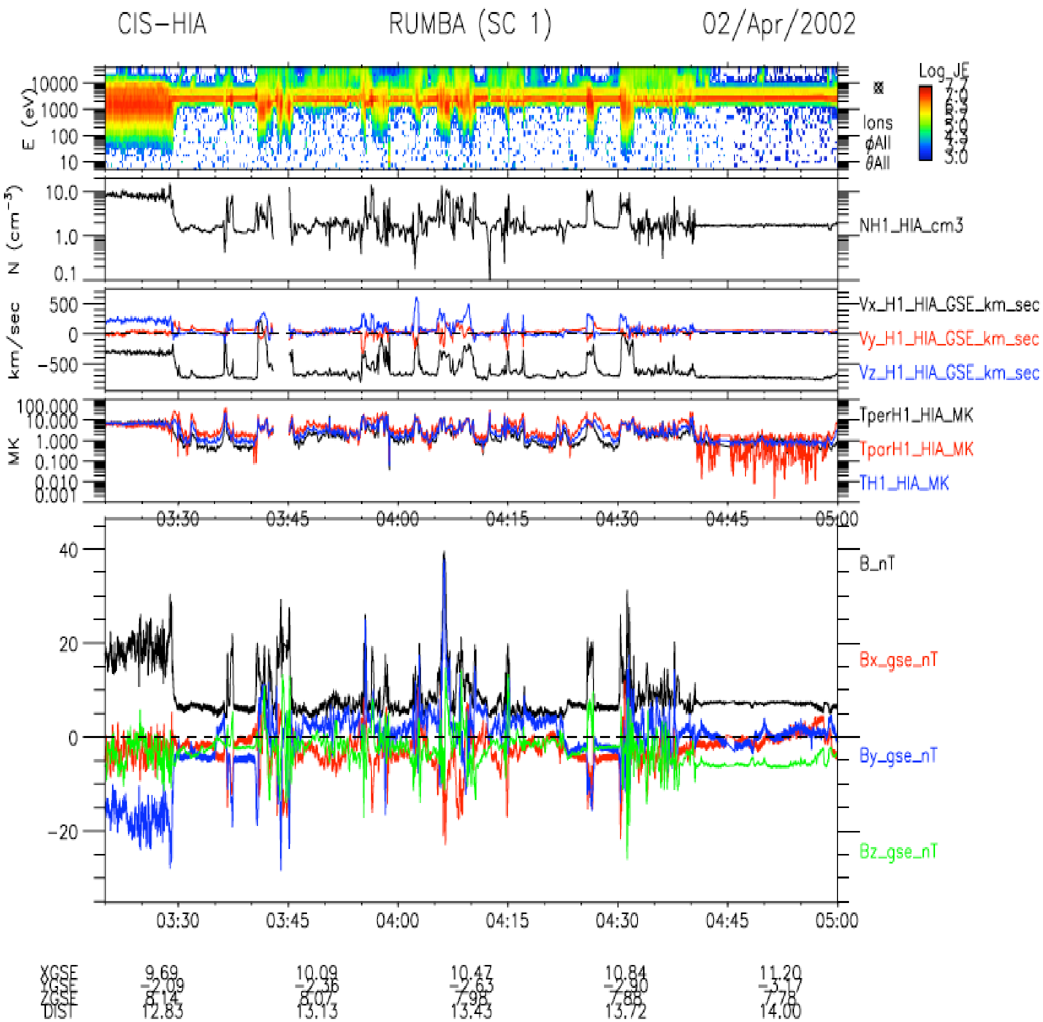
$$n(\mathbf{r}, t) = \int f(\mathbf{r}, \mathbf{v}, t)\, d^3v$$

$$<\mathbf{v}> = \int \mathbf{v}\, f(\mathbf{r}, \mathbf{v}, t)\, d^3v$$

$$<v^2> \int v^2 f(\mathbf{r}, \mathbf{v}, t)\, d^3v$$

$$\boldsymbol{P} = m\int (\mathbf{v}-<\mathbf{v}>)(\mathbf{v}-<\mathbf{v}>)\, f(\mathbf{r}, \mathbf{v}, t)\, d^3v$$

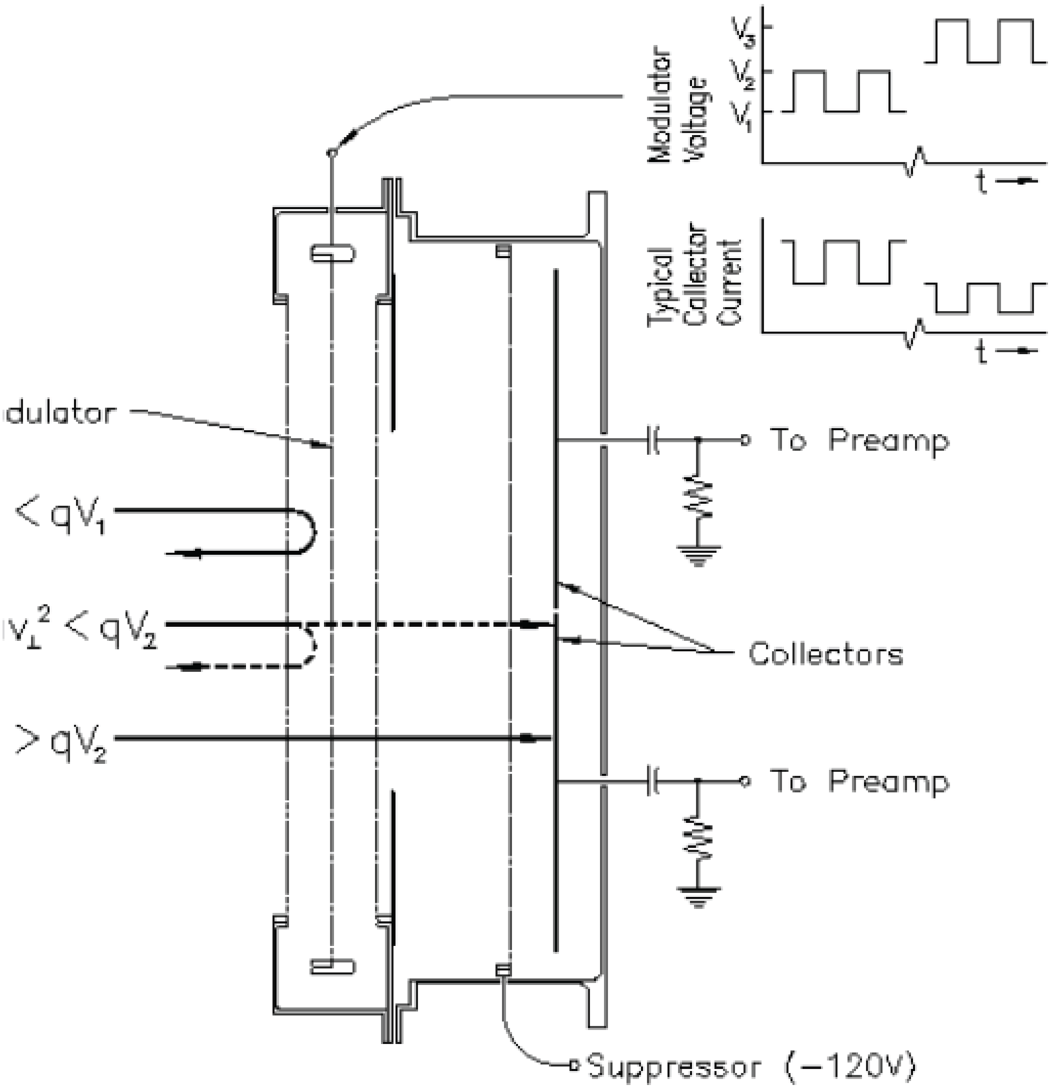
$$\boldsymbol{P} = n\, k\, \boldsymbol{T}$$



- Typical summary Plot from an ESA (Cluster)

- Energy flux (top) and computed Bulk parameters *n*, <***v***> and *T*.

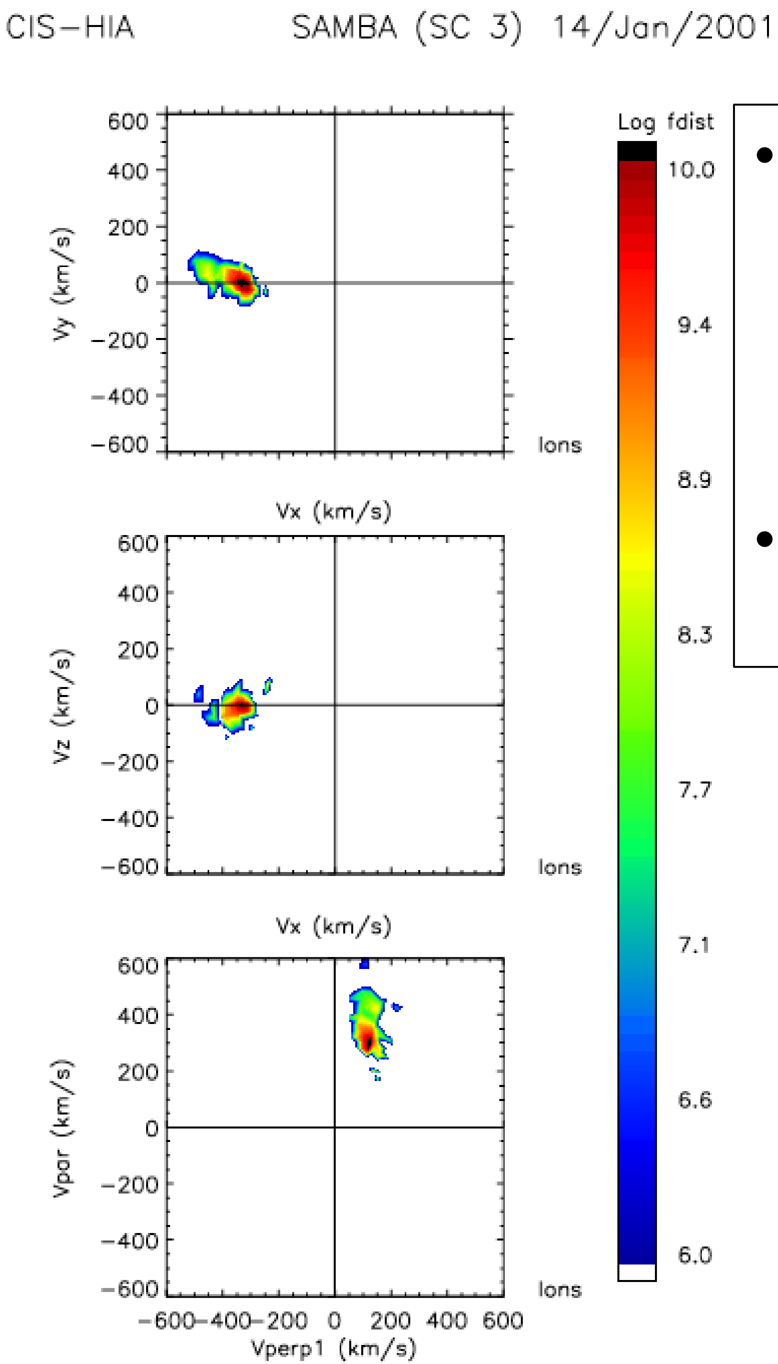
- Magnetic field (bottom)



- Faraday cups measure particles from a few tens of eV to a few keV.

- Faraday Cups rugged, can operate for many years (Voyager)

$$I = eA\int \mathbf{v}\; f(\mathbf{v})\; S(v\})d^3v$$



- ESAs do ***not*** measure ***v***, *q*, or *m*. However, SW data are plotted in velocity space, identifying H^+ and He^{++} ion beams.
- How is information on ***v***, q and *m* of the different particles obtained?

- Energy per charge of H^+ and He^{++} ions (α's) are

H^+ $$(E/q)_+ = m_+ v_+^2/2q_+$$

He^{++} $$(E/q)_\alpha = m_\alpha v_\alpha^2/2q_\alpha = m_+ v_\alpha^2/q_+$$

where $m_\alpha = 4m_+$ and $q_\alpha = 2q_+$.

- Interpretation of ESA data has assumed that

"all particles are traveling at the same mean velocity in steady-state plasmas with a frozen in magnetic field"

Hundhausen, 1968

- If H^+ and He^{++} are *traveling together*, then $v_+ = v_\alpha = V_{sw}$.

For H^+

$$(E/q)_+ = m_+ v_+^2/2q_+ = m_+ V_{sw}^2/2q_+$$

For He^{++}

$$(E/q)_\alpha = m_+ \, v_\alpha^2/q_+ = m_+ V_{sw}^2/q_+$$

Hence,

$$\boxed{(E/q)_\alpha = 2\,(E/q)_+}$$

- Thus, if we *assume* all particles are H^+ in the velocity space, find a beam centered at V_{sw} and identify it as H^+. Another "H^+" beam centered at $(2)^{1/2}V_{sw}$ will be interpreted as He^{++} ions.

- A mass analyzer is needed to identify $\boldsymbol{v}$ and m/q.

Basic Theories and Concepts to interpret space plasma observations:

1. Coupled Lorentz-Maxwell equations (6N equations).
2. Coupled Boltzmann-Maxwell equations (Use distribution function)
3. Coupled Fluid-Maxwell equations (Use macroscopic variables)

- 1 and 2 *are equivalent* for *collisionless plasmas*. Theory is self-consistent and gives a complete picture of space plasma.
- Lorentz-Maxwell approach avoided in the past because analytical solutions not possible.
- Today, the coupled theory used more often because we have super computers to track the particles.
- Most PIC simulations limited to 1 and 2D as computer capability still limited. However, computer capability is continually improving.
- Simulation tools important for data analysis to help interpret complex features.

- MHD fluid equations are conservation equations obtained from the *velocity moments* of the Boltzmann equation. They describe an approximate picture.

Basic theory of space plasmas

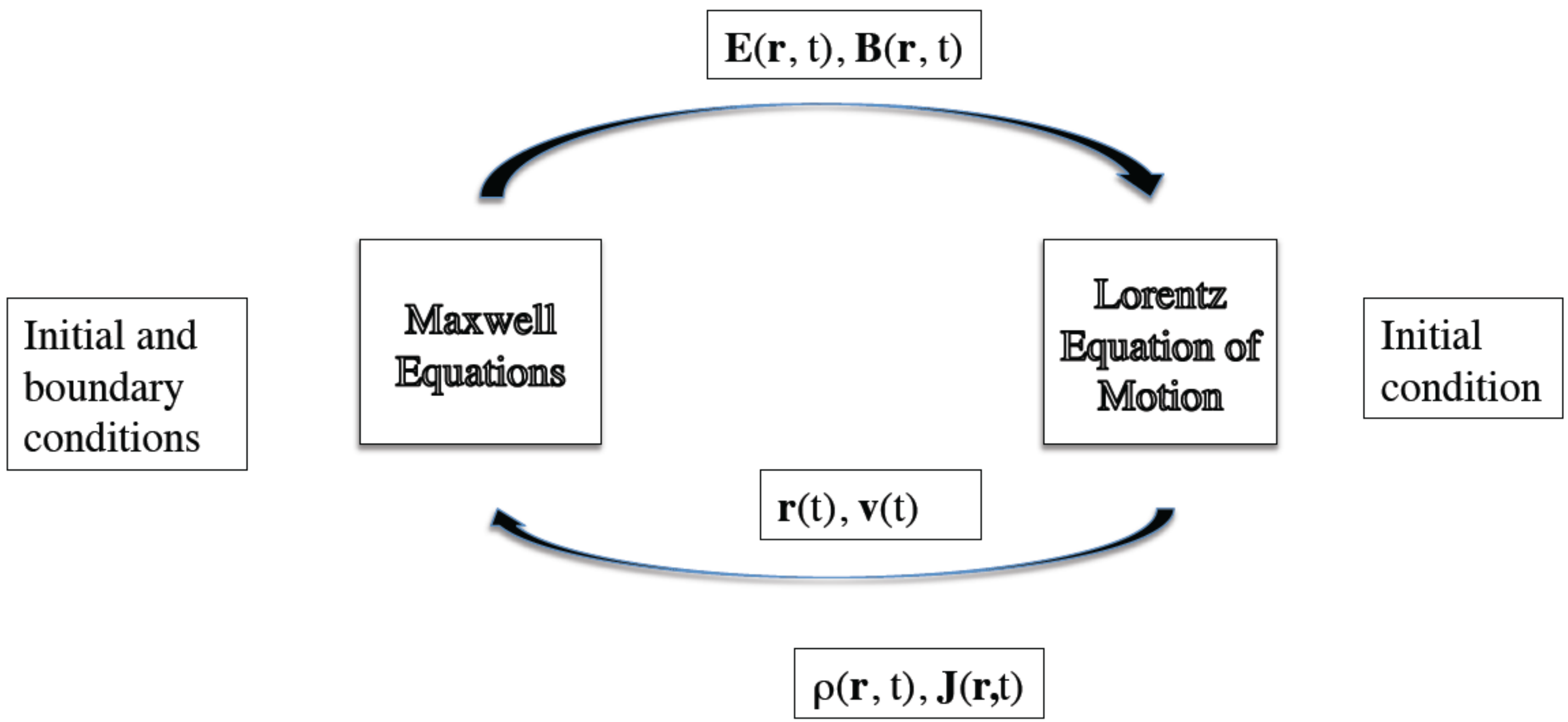
The coupled Lorentz (Boltzmann) and Maxwell equations

$$\frac{d\mathbf{p}_k}{dt} = q_k(\mathbf{E} + \mathbf{v_k} \times \mathbf{B})$$
$$\frac{d\mathcal{E}_k}{dt} = q_k\mathbf{v}_k \cdot \mathbf{E} \quad (6)$$

$$\frac{\partial f(\mathbf{r},\mathbf{v})}{\partial t} + \mathbf{v} \cdot \frac{\partial f \mathbf{r},\mathbf{v})}{\partial \mathbf{r}} + \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f(\mathbf{r},\mathbf{v})}{\partial \mathbf{v}} = 0 \quad (7)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (8)$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \partial \mathbf{D}/\partial t \quad (9)$$
$$\nabla \cdot \mathbf{D} = \rho_c \quad (10)$$
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \quad (11)$$

Self-consistent theory of space plasmas



- *Self-Consistency*: Particle motions produce the required electromagnetic fields that in turn are necessary to create the particle motions.

MHD Equations:

- The first three velocity moments yield *mass, momentum and energy conservation* equations.

- *Advantages*: Reduces the number of variables from 6N to a few macroscopic variables:

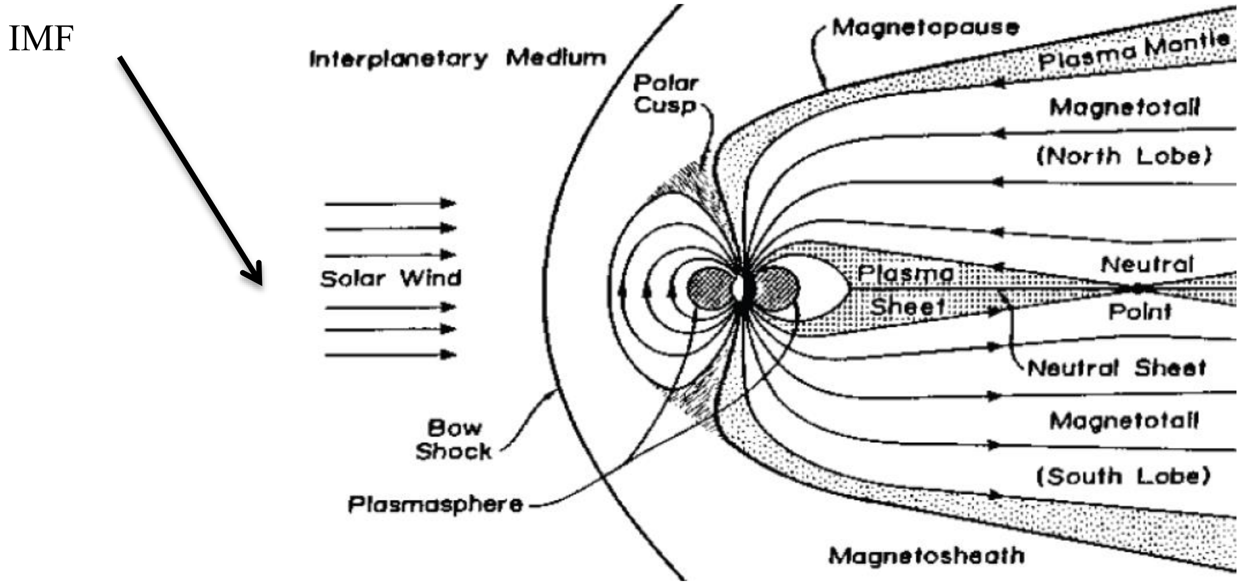
  n, $\mathbf{U}$, T, ….

$$\partial n/\partial t + \nabla\cdot n\mathbf{U} = 0 \qquad (1)$$

$$d\mathbf{U}/dt = -\nabla p + \mathbf{J}\times\mathbf{B} \qquad (2)$$

$$\partial/\partial t\,[nmU^2/2 + p/(\gamma-1) + B^2/2\mu_0] + \nabla\cdot[nmU^2U/2 + \gamma p\mathbf{U}/(\gamma-1) + ExB/\mu_0] = 0 \qquad (3)$$

MHD Description of Solar Wind , IMF, bow shock, and Magnetosphere



- SW *flows* out from the Sun.
- Solar magnetic field *transported* out *frozen* in the SW.
- SW is *supersonic*, hence a *shock wave* forms in front of Earth.
- Magnetosphere formed by the SW confining the geomagnetic field.
- A long tail produced by convecting “connected” IMF-geomagnetic field with the SW.

- MHD equations alone *not sufficient* to describe space plasma behavior self-consistently.

- There are always *more unknowns* than *number of equations*.

- For example, Particle flux conservation: $\partial n/\partial t + \nabla \cdot n\mathbf{U} = 0$, Four unknowns ($n$, $\mathbf{U}$), only three equations.

- Computing higher moments does not solve the problem. *New unknowns* are introduced.

- For a complete MHD description, one needs all velocity moments to solve the closure problem. *Not practical!*

- For a *finite number* of moment equations, MHD equations often supplemented by *Adiabatic equation of state* or *Ohm's law*.

- Adiabatic plasma: *No heat flux*, hence not consistent with many space plasma observations.

- Ohm's law. *No conductivity model exists for collisionless plasmas.*

- To remedy this problem, MHD treats space plasmas as fluid with *infinite conductivity* ($\sigma = \infty$).

  - *Ideal fluids* conserve magnetic flux, leads to frozen-in-field dynamics: *No EMF* is generated.

- Approximation means you throw away information. You need to ask *what and how much physics is lost.*

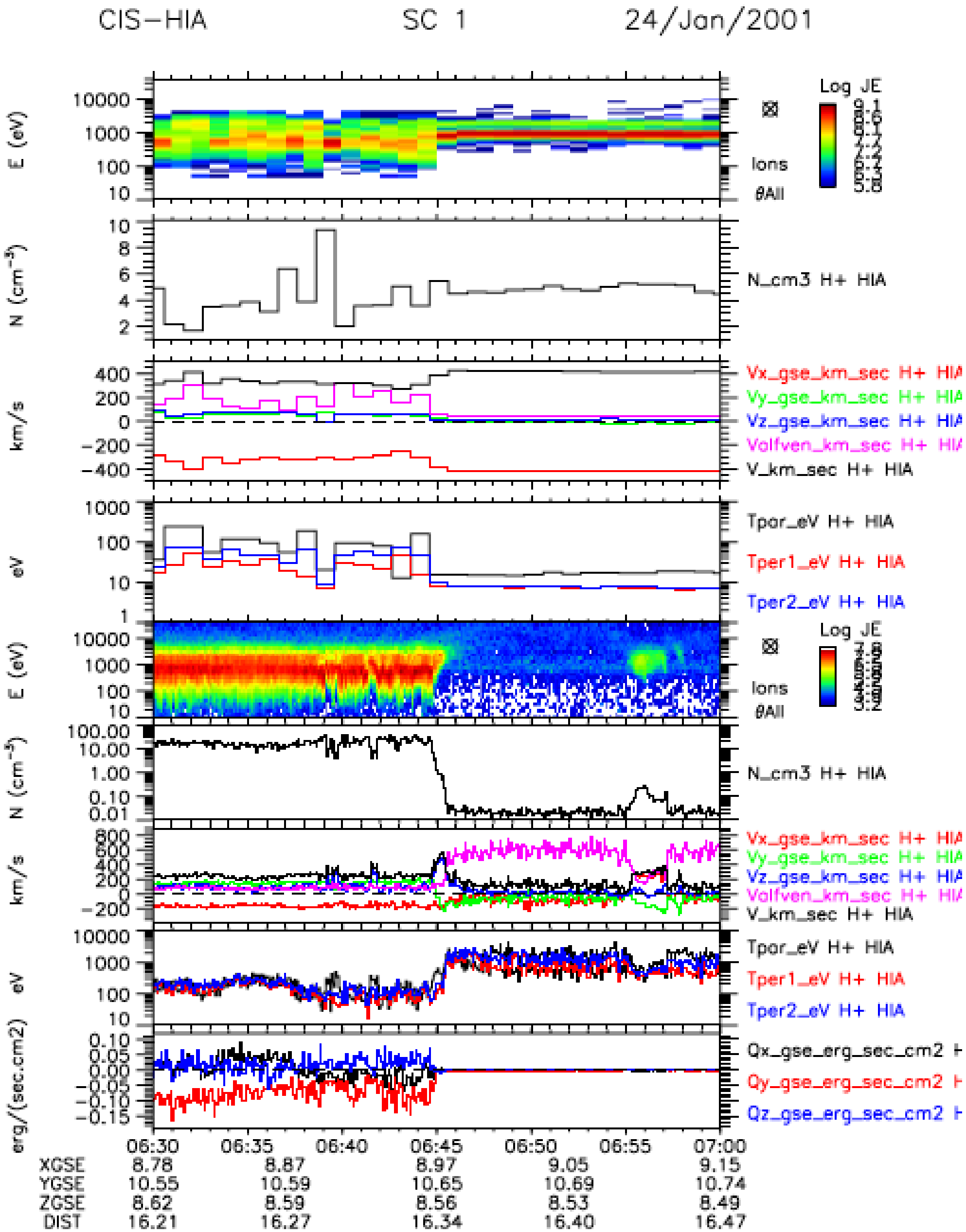
- Observations *not fully explained* by MHD theories and concepts.

- *Solar Wind: Heat flux carried by electrons.*

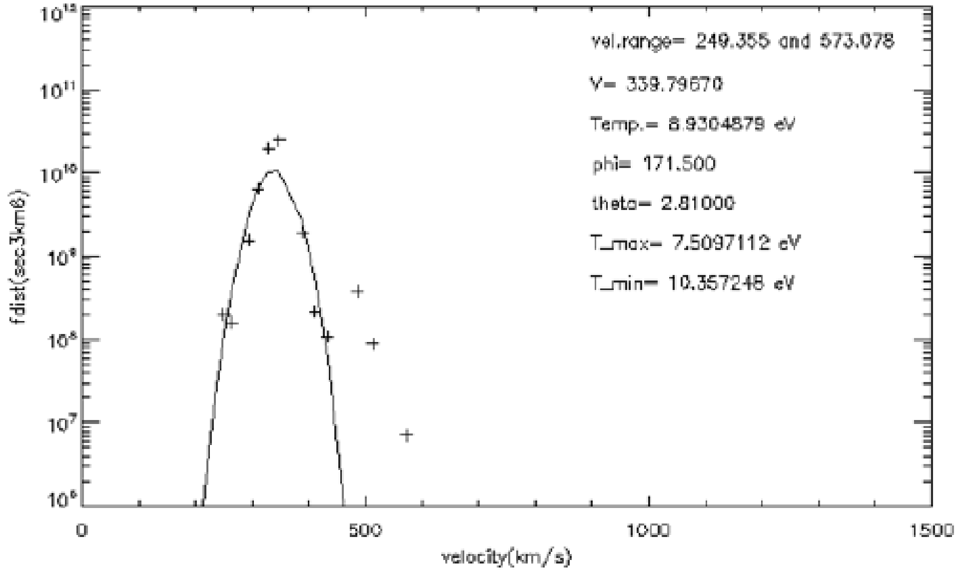
- *Bow shock*: Different from ordinary fluid shocks. Bow shock reflects up to 20% of incident SW back into the upstream region

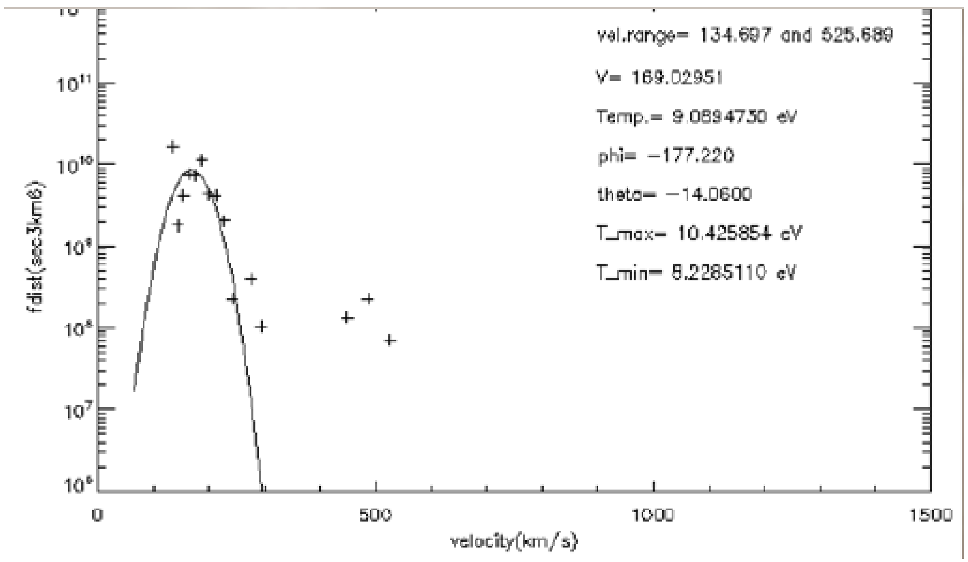
- The remaining 80% transmitted across bow shock is not immediately thermalized.

- The bulk flow in the downstream of bow shock can often remain *super-Alfvenic*.



- Bulk flow remains Super-Alfvénic in Magnetosheath.
- SW is not thermalized at the bow shock.





- SW H^+ beam *slowed down* going across the bow shock but *not* thermalized.
- What shifts down the peak of the SW beam?

Fundamental equation for *Electric Field*.

From $\nabla\times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$ and $\boldsymbol{B} = \nabla\times \boldsymbol{A}$, obtain

$\nabla\times (\boldsymbol{E} - \partial \boldsymbol{A}/\partial t) = 0$. Let $\boldsymbol{E} = -\nabla\varphi$, then

$$\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t \qquad (13)$$

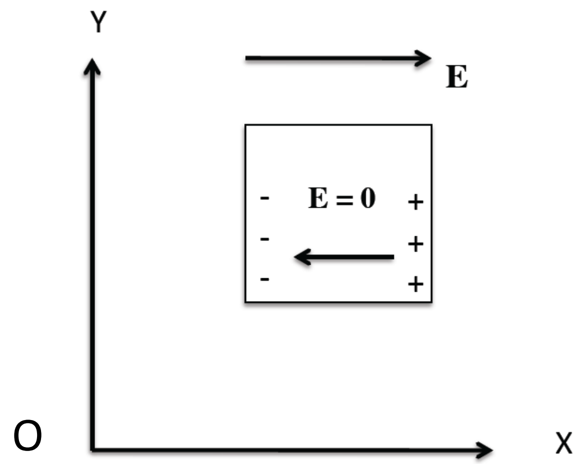
where

$$\phi(\mathbf{r},t) = \frac{1}{4\pi\epsilon_o}\int\frac{(\rho',t)d^3r'}{|\mathbf{r}-\mathbf{r}'|} \qquad (14)$$

and

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_o}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}',t)d^3r'}{|\mathbf{r}-\mathbf{r}'|} \qquad (15)$$

Illustrate how particles in plasmas respond to electric force



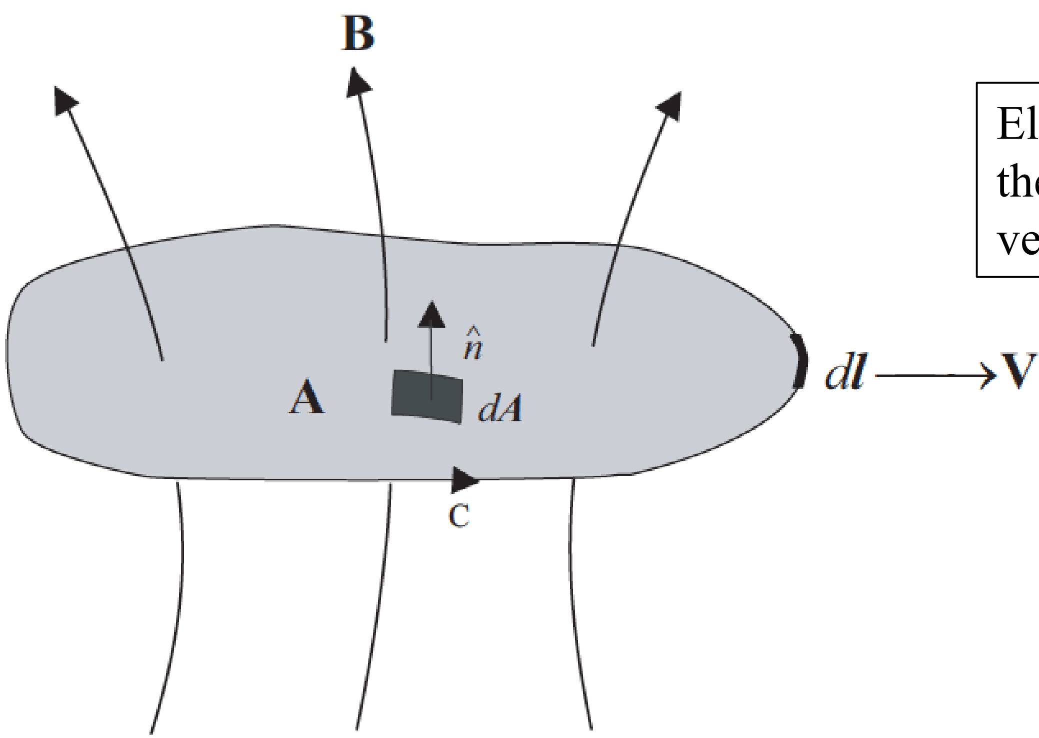
- Let an isolated plasma blob be uniform in space and charge neutral with equal number of protons(p^+) and electrons (e^-). The plasma blob is in equilibrium.
- Apply an **E**-field to a *stationary plasma blob*. No magnetic field, **B**=0
- Inside the plasma blob, the force $q\boldsymbol{E}$ pushes electrons and ions in opposite directions. Produces an **E**-field opposing applied ***E***.
- Motion stops when the *total **force*** on the particles *vanishes*.
- *E = 0 inside equilibrium plasmas*

$$\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$$

- The first term requires free charges in plasma. Plasmas have high *electrical conductivity*. No free charges accumulate so the first term disappears (*Caveat:* Free charges ρ can exist in *double layers)*.

- *Inductive electric fields* responsible for the dynamics of space plasmas.

Faraday's law, one of the most important equations for space plasmas



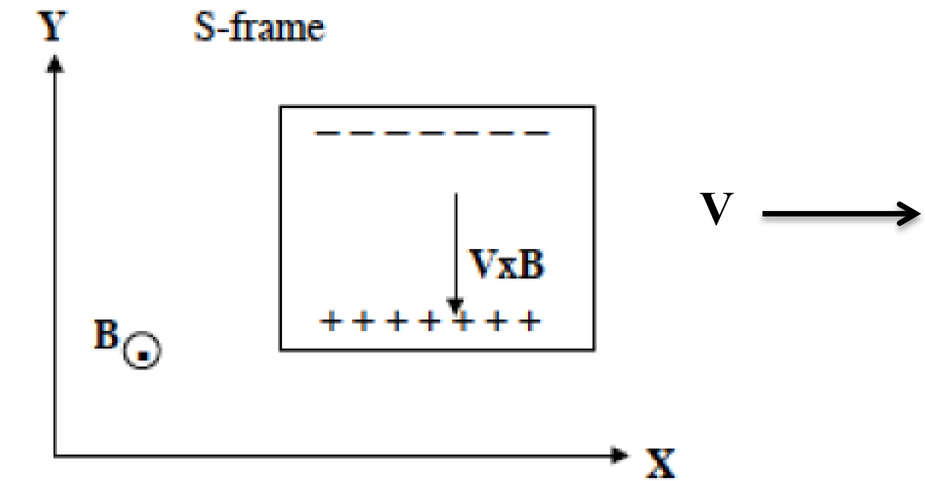
Electric field measured in the frame of the contour element $\boldsymbol{dl}$ moving with velocity $\boldsymbol{V}$ (S' frame).

$$EMF = -d\Phi/dt$$

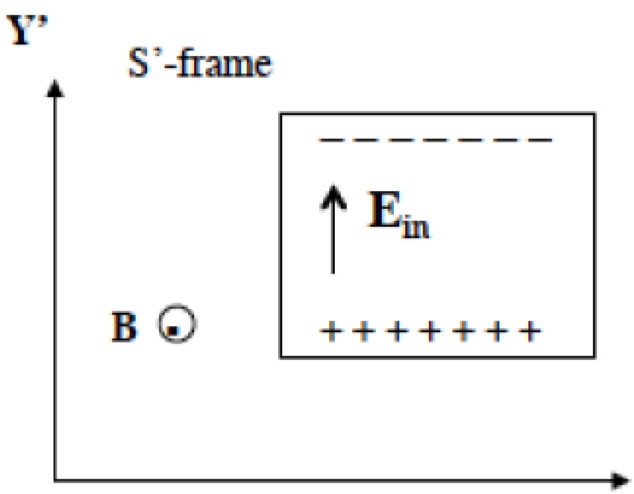
Φ = magnetic flux enclosed by the contour C

$$\oint (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \cdot d\mathbf{l} = \mathbf{EMF}$$

Moving Plasma Blob. $\mathbf{B} \neq 0$, $\mathbf{E} = 0$



V = **V**x
F = **qVxB**
Charge separation
Motion Until **F = q(E+VxB)=0**



V = 0
F = qVxB = 0
Charge separation still there
F = **q**$\mathbf{E}_{in}$

Moving Space plasmas. Physics can be examined in stationary and moving frames. The quantities in different coordinate systems given by Lorentz transformation equations.

Lorentz transformation equations for $\mathbf{E}$ and $\mathbf{B}$ can be written vectorially as

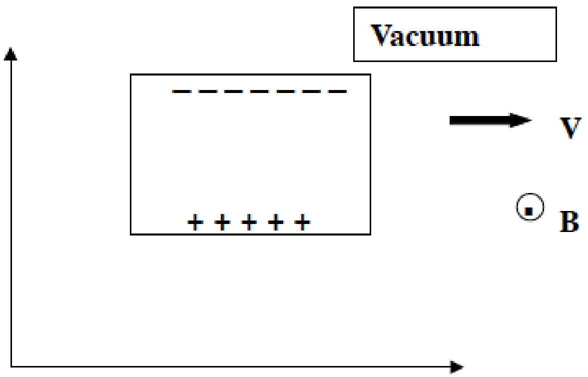
$$\begin{aligned} \mathbf{E}'_{\parallel} &= \mathbf{E}_{\parallel} \\ \mathbf{E}'_{\perp} &= \gamma(\mathbf{E} + \mathbf{V} \times \mathbf{B})_{\perp} \\ \mathbf{B}'_{\parallel} &= \mathbf{B}_{\parallel} \\ \mathbf{B}'_{\perp} &= \gamma(\mathbf{B} - \frac{\mathbf{V}}{c^2} \times \mathbf{E})_{\perp} \end{aligned} \quad (12)$$

The subindices $(\parallel)$ and $(\perp)$ here refer to directions relative to $\mathbf{V}$, velocity of stationary frame relative to moving frame. For non-relativistic situation $(\gamma = 1)$ and to order V/c^2, the magnetic field is the same in the two frames but the electric field has a different expression, $\mathbf{E}' = (\mathbf{E} + \mathbf{V} \times \mathbf{B})$. Hence, when discussing electric fields, the reference frame must be stated.

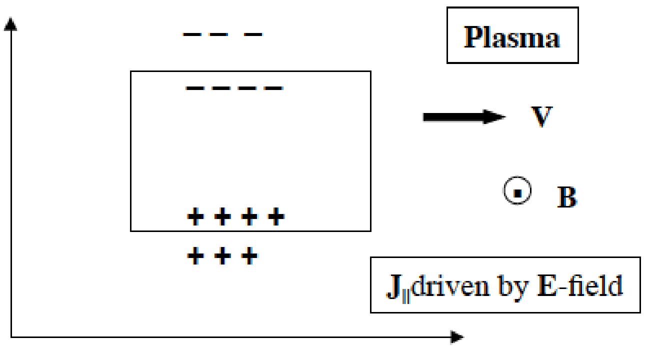
E'_y = -V_x B_z, B'_z = B_z and E_y’ = E_z' = B_x' = B_y'= 0

Motion of plasma blob surrounded by Vacuum (top) or by another plasma (bottom)

Induced Current outside Blob



- Charges pile up at edges
- Form surface Charges
- Inside: **E** = 0
- Outside: **E**≠0



- Surface charges flow out
- Current flows along **B**
- q**V**x**B** > q**E**
- Charges move again
- If **V** constant, steady state **J**
- Current **J** sustained by q**V**x**B** force

The End

Vector Point Function

- The physical variables in Lorentz and Maxwell equation are *vector point functions*.

- Consider a point static charge $q_k\,(\mathbf{r}) = q_k\,\delta(\mathbf{r} - \mathbf{r}_k)$,
  where $\delta(\mathbf{r} - \mathbf{r}_k) = \delta(\mathbf{x} - \mathbf{x}_k)\,\delta(\mathbf{y} - \mathbf{y}_k)\,\delta(\mathbf{z} - \mathbf{z}_k)$.

- *Electric field* produced by this charge given by Coulomb's law
  $$\mathbf{E}(\mathbf{r}) = q_k\,(\mathbf{r} - \mathbf{r}_k)/\,|(\mathbf{r} - \mathbf{r}_k)|^3.$$

- Define Electric field at $\mathbf{r}$ as *Force per unit charge*
  $$\mathbf{E}\,(\mathbf{r}) = \lim_{q\to 0} \mathbf{F}_E/q$$

- $\mathbf{E}$ is parallel to $\mathbf{F}_E$ and the charge q is accelerated in the direction $\mathbf{E}$. If there are many charges, $\mathbf{E}\,(\mathbf{r}) = \Sigma\, q_k\,(\mathbf{r} - \mathbf{r}_k)/|(\mathbf{r} - \mathbf{r}_k)|$.

- A set of point charges = charge density $\rho\,(\mathbf{r}) = \Sigma qk\,\delta(\mathbf{r} - \mathbf{r}_k)$. Then
  $$\mathbf{E}(\mathbf{r}) = \int d^3r\,\rho\,(\mathbf{r})\,(\mathbf{r} - \mathbf{r}_k)/\,|(\mathbf{r} - \mathbf{r}_k)|^3.$$

- If a charge q is moving with velocity **v**, there is now a *current,* q**v**, which gives rise to a magnetic field **B** at that point.

- In the presence of a magnetic field, a charge executes a circular motion due to the magnetic force, $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$.

- Magnitude of $\mathbf{F}_B$ depends on the magnitude and direction of **v** and **B** can be defined as *force per unit current*.

- Force is maximum when **v** is perpendicular to **B** and minimum when parallel to **B**. The intensity of the magnetic field in terms of the maximum force $|\mathbf{F}_B|$ max is

$$|\mathbf{B}| = \lim_{qv \to 0} |\mathbf{F}_B|_{max}/q\mathbf{v}$$

- The direction of B is defined as the direction in which q would move when it experience no magnetic force.

- Continuous distribution of current, use Biot-Savat's law,

$$\mathbf{B}(\mathbf{r}) = \int d^3r\, \mathbf{J}(\mathbf{r})\, (\mathbf{r} - \mathbf{r}_k)/\, |(\mathbf{r} - \mathbf{r}_k)|^3.$$

- In the frame moving with the charge **v**, **J** vanishes. But q is there and so is **E**-field. One can thus look at ***E*** *as primary quantity* and ***B*** *is consequence of q in motion*.